Discount-Rate Shocks, Price-Rent Variations, and Business Cycles[1]

Jianjun Miao[a] Pengfei Wang[b] Tao Zha[c]

[a]Boston University

[b]Hong Kong University of Sciences and Technology

[c]Federal Reserve Bank of Atlanta and NBER

5th Workshop in Macro Banking and Finance
Università Cattolica del Sacro Cuore
September 14-15, 2017

Introduction

- The collapse of real estate prices is a key factor contributing to the 2008 financial crisis.
- Macroeconomic models (e.g., Iacoviello (2005), Iacoviello and Neri (2010), and Liu, Wang, and Zha (2013)) have emphasized housing demand shocks that drive the comovement between real estate prices and business cycles, especially between real estate prices and investment fluctuations.
- The micro evidence provided by Chaney, Sraer, and Thesmar (2012) also finds that, over the 1993-2007 period, an increase in commercial real estate value enabled a representative U.S. corporate firm to raise investment significantly.
- Their micro evidence suggests that shocks to commercial real estate prices may have important effects on aggregate investment.

Introduction

- But one serious problem with housing demand shocks emphasized by the existing macroeconomic models is their failture in generating the large variations of the price-rent ratio, as these shocks drive the price and rent fluctuations in a similar magnitude.
- Consequently, it also fails to generate the long-term prediction of returns on real estate by the price-rent variations.
- We need a different shock and a different mechanism in macroeconomic models to link these key asset-pricing properties with business cycles.

Facts about commercial real estate prices

- We first document several key facts about commercial real estate prices that cannot be accounted for by the existing macroeconomic models.
  - Commercial real estate prices are often approximated by home prices in the studies on investment (e.g., Chaney, Sraer, and Thesmar (2012)).
  - We construct the time series of commercial real estate prices directly, which is highly correlated with the home price series.

1. The most important fact is the observed large variation of the ratio of price to rent (the valuation ratio).
2. The real estate price is significantly more volatile than the rent and the real economy.
3. Price-rent variations have a power in predicting long-term returns of real estate (Cochrane, 2011; Ghysels et al., 2013).
4. The price-rent ratio comoves with the business cycle.

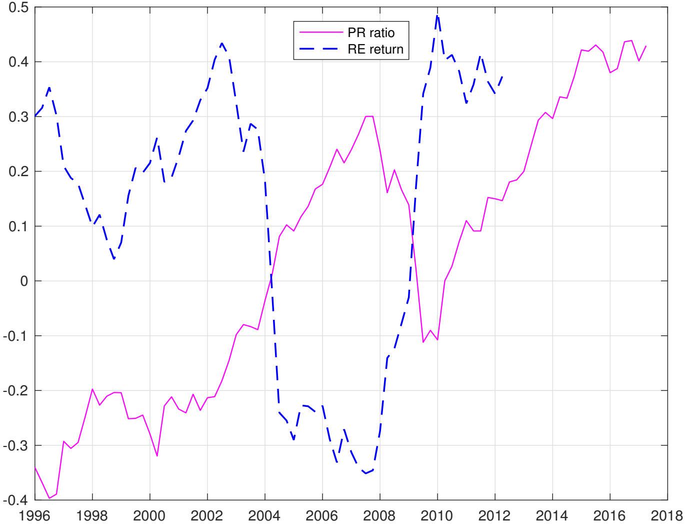
Financial and real volatilities (%)

Description	Volatility	Data
Investment	$std\,(\Delta \log I_t)$	1.679
Output	$std\,(\Delta \log Y_t)$	0.697
Consumption	$std\,(\Delta \log C_t)$	0.444
Rental price	$std\,(\Delta \log R_{ct})$	1.245
Real estate price	$std\,(\Delta \log p_t)$	4.171
Price-rent	$std\,(\Delta \log(p_t/R_{ct}))$	3.909

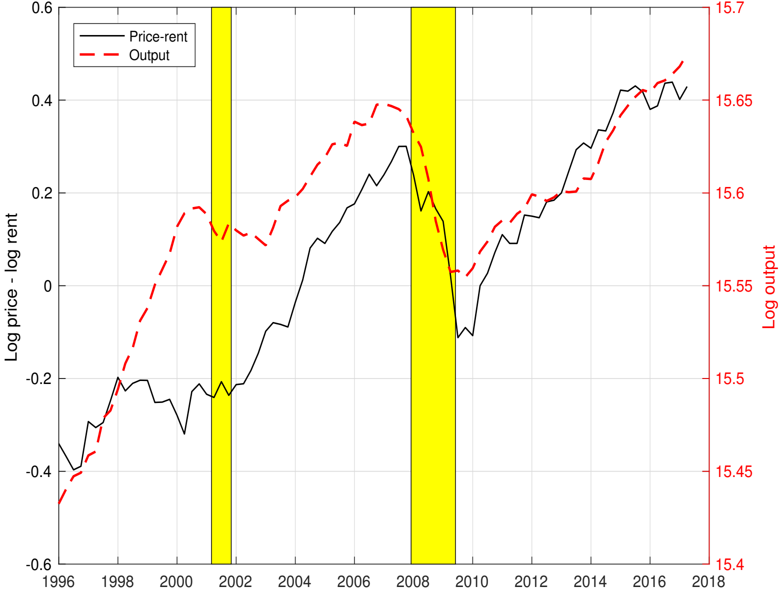
Prediction of real estate returns by the rent-to-price ratio at different horizons

Predictive regression: $r_{t\to t+k} = \alpha_0 + \alpha_1 \log(R_{ct}/p_t) + \varepsilon_{t+k}$		
Horizon Quarter (year)	Data (α_1)	Data (R^2)
8 (2)	0.20 (0.07, 0.33)	0.08
12 (3)	0.37 (0.20, 0.54)	0.15
16 (4)	0.58 (0.39, 0.78)	0.26
20 (5)	0.77 (0.58, 0.96)	0.40
24 (6)	0.82 (0.65, 1.00)	0.50

Note: We report the OLS estimates of α_1 and R^2. The numbers in parentheses in the column headed by "Data (α_1)" represent the 90% confidence interval of the estimated coefficient. The real estate return from t to $t+k$ is defined as $r_{t\to t+k} = \log(p_{t+k}/p_t)$.



"PR" stands for the price-rent ratio and "RE" stands for real estate. The PR ratio is in log value and scaled to make it visually comparable with the returns. The real estate return is over the five-year horizon. The price-rent ratio is in log at time t.



Price-rent
Output
Log price - log rent
Log output
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
15.7
15.65
15.6
15.55
15.5
15.45
15.4
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018

What do we do in this paper?

- We develop and estimate a general-equilibrium model that embodies three essential ingredients:
  - discount-rate shocks as emphasized by Albuquerque et al. (2016),
  - collateral constraints on firms (Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999),
  - and the endogenous TFP mechanism (Moll, 2014).
- The model links the real estate market with the real economy and accounts for the following key facts.
  - The real estate price fluctuates more than the rental price and the business cycle.
  - Price-rent variations have a power in predicting long-term returns of real estate.
  - The price-rent ratio comoves with the business cycle.

Discount-rate shocks

- Both macro and finance literatures have modeled discount-rate or time-preference shocks (e.g., Eggertsson and Woodford (2003), Cochrane (2011), Albuquerque et al. (2016), and Hall (2017)).
- Discount-rate shocks are used in the finance literature to generate key asset-pricing properties such as volatility and long-term predictability.
- The finance literature focuses on the endowment economy.
- When the production economy is introduced in the macroeconomic literature, the asset-pricing implications of discount-rate shocks are often unexplored and the discount-rate shock tends to generte the opposite movements between consumption and investment (as agents desire to save more and consume less in response to a positive discount-rate shock).
- For macroeconomic models, we show that the collateral constraint and endogenous TFP are essential for mitigating the opposite movements between consumption and investment and at the same time accounting for the volatility and long-term predictability in the real estate market.

The production-economy model

- The representative household maximizes its utility and accumulates physical capital.
- There are a variety of intermediate goods and each good is produced by a continuum of identical producers.
- There are a continuum of heterogeneous final-goods firms indexed by idiosyncratic productivity shocks.
- Final-goods firms trade real estate properties and rent out commercial real estate to intermediate-goods producers.
- Final-goods firms borrow against their real estate value to finance working capital.

Household

- Maximizes the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \Theta_t \beta^t \left[\log \left(C_t - \gamma C_{t-1} \right) - \psi_t \frac{N_t^{1+\nu}}{1+\nu} \right],$$

- We follow Albuquerque et al. (2016) and introduce the discount-rate shock Θ_t, which has become a key shock capable of explaining the real estate price volatility and the business cycle.
- The variables $\theta_t \equiv \Theta_t / \Theta_{t-1}$ and ψ_t are exogenous shocks to the discount rates and labor supply:

$$\log \theta_t = (1 - \rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \sigma_\theta \varepsilon_{\theta,t},$$
$$\log \psi_t = (1 - \rho_\psi) \log \psi + \rho_\psi \log \psi_{t-1} + \sigma_\psi \varepsilon_{\psi,t}.$$

where $\varepsilon_{\theta,t}$ and $\varepsilon_{\psi,t}$ are i.i.d. standard normal random variables.

Household

- The household chooses consumption C_t, investment I_t, the capital utilization rate u_t, and bonds B_{t+1}, subject to the intertemporal budget constraint

$$C_t + \frac{I_t}{Z_t} + \frac{B_{t+1}}{R_{ft}} \leq w_t N_t + u_t R_{kt} K_t + D_t + B_t,$$

- The variable Z_t represents an aggregate investment-specific technology shock that has both permanent and transitory components (Greenwood, Hercowitz, and Krusell, 1997) and (Krusell, Ohanian, Ríos-Rull, and Violante, 2000):

$$Z_t = Z_t^p v_{zt}, \;\; Z_t^p = Z_{t-1}^p g_{zt},$$

$$\log g_{zt} = (1 - \rho_z) \log g_z + \rho_z \log(g_{z,t-1}) + \sigma_z \varepsilon_{zt},$$

$$\log v_{zt} = \rho_{v_z} \log v_{z,t-1} + \sigma_{v_z} \varepsilon_{v_z,t}.$$

- Investment is subject to quadratic adjustment costs (Christiano, Eichenbaum, and Evans, 2005):

$$K_{t+1} = (1 - \delta(u_t))K_t + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right] I_t.$$

Intermediate-goods producers

- There is a continuum of intermediate-goods producers.
  - Each intermediate good $j \in [0,1]$ is produced by a continuum of identical competitive producers of measure unity.
  - The representative producer owns a constant-returns-to-scale technology to produce good j
    - ★ by hiring labor $N_t(j)$, renting real estate property H_t, and renting capital $K_t(j)$ from households.
  - Thus, the producer's decision problem becomes

$$\max_{N_t(j), H_t(j), K_t(j)} P_{Xt}(j) X_t(j) - w_t N_t(j) - R_{ct} H_t(j) - R_{kt} K_t(j),$$

    - ★ where $X_t(j) \equiv A_t \left[K_t^{1-\phi}(j) H_t^{\phi}(j) \right]^{\alpha} N_t^{1-\alpha}(j)$ and $P_{Xt}(j)$ represents the competitive price of good j.
- The aggregate technology shock A_t consists of both permanent and transitory components (Krusell et al., 2000; Aguiar and Gopinath, 2007)

$$A_t = A_t^p \nu_{at}, \; A_t^p = A_{t-1}^p g_{at},$$

where the exogenous technology processes are

$$\log g_{at} = (1 - \rho_a) \log g_a + \rho_a \log(g_{a,t-1}) + \sigma_a \varepsilon_{at},$$
$$\log \nu_{at} = \rho_{v_a} \log \nu_{a,t-1} + \sigma_{v_a} \varepsilon_{v_{at}}.$$

Final-goods firms

There is a continuum of heterogeneous competitive firms.

- Each firm $i \in [0,1]$ combines a variety of intermediate goods $x_t^i(j)$ to produce final consumption goods with the aggregate production technology

$$y_t^i = a_t^i \exp\left(\int_0^1 \log x_t^i(j) dj\right),$$

where a_t^i represents an idiosyncratic productivity shock drawn independently and identically from a fixed distribution with pdf $f\left(a_t^i\right)$ and cdf $F\left(a_t^i\right)$ on the $(0,\infty)$ support.

  - Firm i purchases intermediate good j at the price $P_{Xt}(j)$.
  - The total spending on working capital is $\int_0^1 P_{Xt}(j) x_t^i(j) dj$.
  - The firm finances its working capital with the standard credit constraint:

$$\int_0^1 P_{Xt}(j) x_t^i(j) dj \leq \lambda p_t h_t^i.$$

  - One could introduce an exogenous shock to the credit constraint, but it is shown by Liu, Wang, and Zha (2013) and Kaplan, Mitman, and Violante (2016) that this shock cannot generate the variation and persistence of real estate price that match the data.

Final-goods firms

Firm i buys and sells real estate property as well as rents h_t^i out to intermediate-goods producers.

- The firm's incomes come from profits and rents.
- Its flow-of-funds constraint is given by

$$d_t^i + p_t(h_{t+1}^i - h_t^i) = y_t^i - \int_0^1 P_{Xt}(j) x_t^i(j) dj + R_{ct} h_t^i, \;\; t \geq 0, \text{with } h_0^i \text{ given.}$$

- Firm i maximizes the discounted present value of dividends

$$\max E_0 \sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t}{\Lambda_0} d_t^i,$$

  where d_t^i denotes dividends and $\beta^t \Lambda_t / \Lambda_0$ is the household's stochastic discount rate.
- The marginal utility of consumption is

$$\Lambda_t = \frac{\Theta_t}{C_t - \gamma C_{t-1}} - \beta \gamma E_t \frac{\Theta_{t+1}}{C_{t+1} - \gamma C_t}.$$

Equilibrium

- The markets clear for real estate, government bonds, and intermediate goods:

$$\int_0^1 h_t^i di = \int H_t(j) dj = 1, B_t = 0,$$

$$\int_0^1 x_t^i(j) di = X_t(j) = A_t \left[K_t^{1-\phi}(j) \, H_t^{\phi}(j) \right]^{\alpha} N_t^{1-\alpha}(j) .$$

- Since the equilibrium is symmetric for intermediate-goods producers, we have

$$P_{Xt}(j) = P_{Xt}, \; H_t(j) = H_t, \; N_t(j) = N_t, \; K_t(j) = u_t K_t,$$

$$X_t(j) = X_t = A_t \left[(u_t K_t)^{1-\phi} H_t^{\phi} \right]^{\alpha} N_t^{1-\alpha}$$

  for all j. The household's dividend income and aggregate output are

$$D_t = \int_0^1 d_t^i di \text{ and } Y_t = \int_0^1 y_t^i di.$$

Equilibrium

A competitive equilibrium consists of price sequences

$$\{w_t, R_{ct}, R_{kt}, p_t, R_{ft}, P_{Xt}\}_{t=0}^{\infty}$$

and allocation sequences

$$\{C_t, I_t, u_t, N_t, Y_t, B_{t+1}, K_{t+1}, X_t, D_t\}_{t=0}^{\infty}$$

such that (a) given the prices, the allocations solve the optimizing problems for households, intermediate-goods producers, and final-goods firms and (b) all markets clear.

Solving firm i's problem

- We first derive the unit cost of production by define the total cost of producing y_t^i as

$$\Phi(y_t^i, a_t^i) \equiv \min_{x_t^i(j)} \quad \int P_{Xt}(j) x_t^i(j) dj,$$

subject to $a_t^i \exp\left(\int \log x_t^i(j) dj\right) \geq y_t^i$.
- Cost-minimization implies that

$$\Phi(y_t^i, a_t^i) = y_t^i \frac{a_t^*}{a_t^i},$$

where the average cost a_t^* is given by

$$a_t^* \equiv \exp\left[\int_0^1 \log P_{Xt}(j) dj\right].$$

- Denote

$$x_t^i = \exp\left(\int_0^1 \log x_t^i(j) dj\right).$$

Proposition 1

The optimal output:

$$y_t^i = \begin{cases} \lambda \frac{a_t^i}{a_t^*} p_t h_t^i & \text{if } a_t^i \geq a_t^* \\ 0 & \text{otherwise} \end{cases},$$

where the average cost a_t^* and aggregate output Y_t are determined jointly by the two simultaneous equations:

$$\lambda \frac{p_t}{a_t^*} \int_{a_t^*}^{\infty} af(a)da = Y_t, \qquad (1)$$

and

$$Y_t = A_t \left(u_t K_t\right)^{\alpha(1-\phi)} H_t^{\alpha\phi} N_t^{1-\alpha} \left[\frac{1}{1-F(a_t^*)} \int_{a_t^*}^{\infty} af(a)da\right], \qquad (2)$$

where the term $\left[\frac{1}{1-F(a_t^*)} \int_{a_t^*}^{\infty} af(a)da\right]$ is endogenous TFP.

Proposition 2

- The asset pricing equation for commercial real estate is

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[R_{ct+1} + p_{t+1} + \lambda p_{t+1} \int_{a^*_{t+1}}^{\infty} \frac{a - a^*_{t+1}}{a^*_{t+1}} f(a) da \right], \quad (3)$$

  where the average credit yield in the next period is

$$\int_{a^*_{t+1}}^{\infty} \frac{a - a^*_{t+1}}{a^*_{t+1}} f(a) da.$$

  - For the productive firms ($a^i_{t+1} \geq a^*_{t+1}$), the above term reflects the average profit generated by one-dollar credit.

- The rental price of real estate is determined by

$$R_{ct} = \frac{\alpha \phi Y_t}{\frac{1}{1-F(a^*_t)} \int_{a^*_t}^{\infty} \frac{a}{a^*_t} f(a) da}. \quad (4)$$

Labor market

- To derive the labor-market curve, we use the labor supply equation

$$\frac{\Lambda_t}{\Theta_t} w_t = \psi_t N_t^{\nu}$$

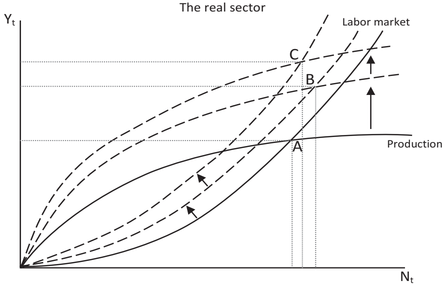
and the labor demand equation

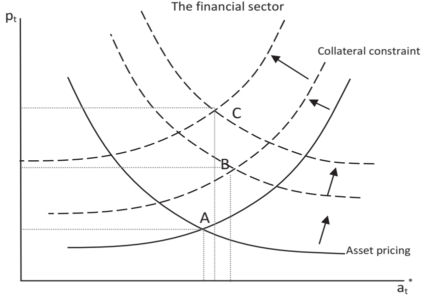
$$(1-\alpha) Y_t = \frac{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}{1 - F(a_t^*)} w_t N_t$$

to eliminate w_t.

- We then obtain the labor-market equation

$$N_t^{1+\nu} = \frac{1 - F(a_t^*)}{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da} \frac{(1-\alpha)\, Y_t \frac{\Lambda_t}{\Theta_t}}{\psi_t}. \tag{5}$$





An illustration of the propagation mechanism: the production and labor-market equations: (2) and (5); the asset-pricing and collateral-constraint equations: (3) and (1).

Fitting the model to the data

- We take the Bayesian approach and fit the log-linearized model to the five key U.S. time series over the period from 1995:Q2 to 2017:Q2:
  - the quality-adjusted relative price of investment,
  - real per capita consumption,
  - real per capita investment (in consumption units),
  - per capita hours worked,
  - the price-rent ratio in commercial real estate.
- The repeated sale price of commercial real estate is available from 1996Q2 on.
  - We allow four lags in our estimation.
  - The sample including four lags begins 1995Q2.

Notation

- We denote a log-linearized variable by $\hat{x}_t \equiv \log \tilde{x}_t - \log \tilde{x}$, where $\tilde{x}$ is the steady state value of the stationary variable $\tilde{x}_t$. For example, $\hat{Y}_t \equiv \log \tilde{Y}_t - \log \tilde{Y}$, $\hat{p}_t \equiv \log \tilde{p}_t - \log \tilde{p}$, $\hat{\theta}_t \equiv \log \theta_t - \log \theta$, and $\hat{a}^*_t \equiv \log a^*_t - \log a^*$.
- The parameter $\mu = \frac{\int_{a^*}^{\infty} \frac{a}{a^*} f(a)\, da}{1-F(a^*)} - 1$ measures the markup, which is estimated to be around 5%.
- Log-linearizing the endogenous TFP $\left[\frac{1}{1-F(a_t^*)} \int_{a_t^*}^{\infty} af(a)da\right]$ gives $\chi \left(\lambda \hat{p}_t - \hat{Y}_t\right)$, where

$$\chi = \frac{a^* + \int_{a^*}^{\infty} af(a)da}{[1 - F(a^*)] \left[1 - F(a^*) + a^* + \int_{a^*}^{\infty} af(a)da\right]}.$$

- The collateral elasticity χ measures how strongly TFP responds to the collateral constraint.

Structural parameters

		Posterior estimates		
Parameter	Representation	Mode	Low	High
ν	Inv Frisch elasticity	0.343	0.088	1.100
χ	Collateral elasticity	0.045	0.044	0.045
δ''/δ'	Capacity utilization	0.850	0.676	1.243
γ	Habit formation	0.558	0.480	0.634
Ω	Capital adjustment	0.245	0.164	0.386

Prior

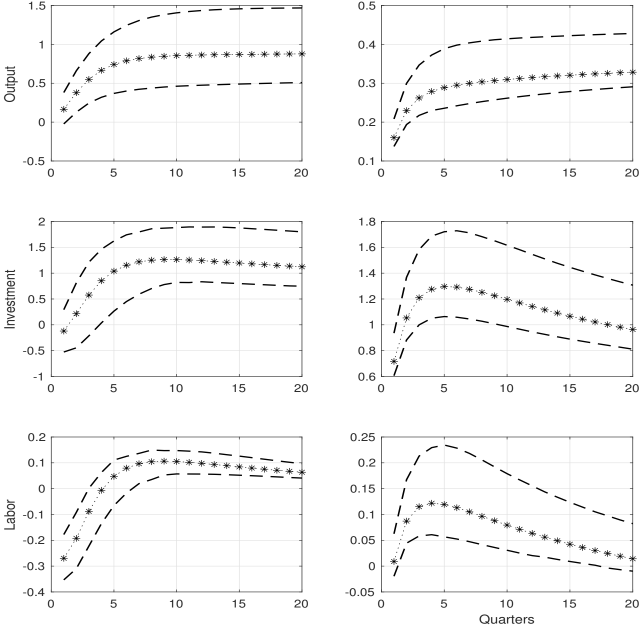
Parameter	Distribution	Settings			
		Low	High	$\bar{\alpha}$	$\bar{\beta}$
ν	Gamma	0.017	1.0	1.0	3.0
χ	Gamma	0.002	0.1	1.0	30.0
δ''/δ'	Gamma	0.1	0.5	4.6	17.2
γ	Beta	0.025	0.775	1.0	2.0
Ω	Gamma	0.1	6.0	1.0	0.5

"Low" and "High" represent the intervals for 0.90 equal-tail probability. The hyperparameters $\bar{\alpha}$ and $\bar{\beta}$ correspond to each prior distribution. The shock standard deviation prior is of inverse-gamma with the 0.9 interval set between 0.0001 and 2.0. This range is wide enough to cover various standard deviation values.

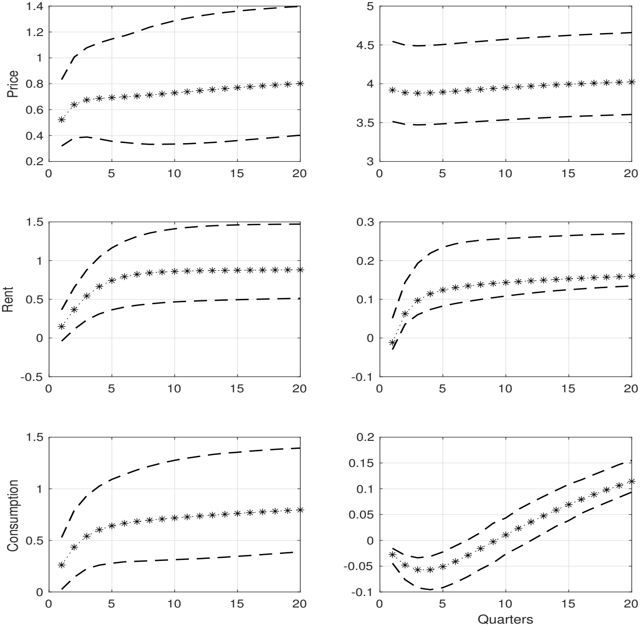
Shock processes

Parameter	Representation	Posterior estimates		
		Mode	Low	High
ρ_z	Permanent investment tech	0.0941	0.0274	0.2765
ρ_{ν_z}	Stationary investment tech	0.0000	0.0114	0.4779
ρ_a	Permanent neutral tech	0.5664	0.4294	0.7403
ρ_{ν_a}	Stationary neutral tech	0.8211	0.7560	0.8835
ρ_θ	Discount rates	0.9994	0.9986	0.9997
ρ_ψ	Labor supply	0.9941	0.9838	0.9967
σ_z	Permanent investment tech	0.0053	0.0044	0.0059
σ_{ν_z}	Stationary investment tech	0.0001	0.00007	0.0019
σ_a	Permanent neutral tech	0.0027	0.0019	0.0038
σ_{ν_a}	Stationary neutral tech	0.0087	0.0078	0.0108
σ_θ	Discount rates	0.0002	0.00018	0.0003
σ_ψ	Labor supply	0.0080	0.0065	0.0124

The prior for all shock AR(1) persistence parameters follows the beta distribution with the hyperparmaters set at $\bar{\alpha} = 1.0$ and $\bar{\beta} = 5.0$. This prior favors strong stationarity.



Impulse responses to a one-standard-deviation shock to neutral technology growth (left panel) and to discount rates (right panel). The star line represents the estimated response. The dashed lines represent the 0.90 probability error bands.



Impulse responses to a one-standard-deviation shock to neutral technology growth (left panel) and to discount rates (right panel). The star line represents the estimated response. The dashed lines represent the 0.90 probability error bands.

Discount-rate shocks: prediction of real estate returns by the rent-to-price ratio at different horizons

Predictive regression: $r_{t\to t+k} = \alpha_0 + \alpha_1 \log(R_{ct}/p_t) + \varepsilon_{t+k}$						
Horizon	Data (α_1)	Model (α_1)	Data (R^2)	Model (R^2)		
Quarter (year)		Median		Median	Low	High
8 (2)	0.20 (0.07, 0.33)	0.37	0.08	0.20	0.04	0.39
12 (3)	0.37 (0.20, 0.54)	0.55	0.15	0.30	0.07	0.52
16 (4)	0.58 (0.39, 0.78)	0.70	0.26	0.38	0.09	0.62
20 (5)	0.77 (0.58, 0.96)	0.82	0.40	0.42	0.11	0.69
24 (6)	0.82 (0.65, 1.00)	0.89	0.50	0.51	0.13	0.74

Note: We report the OLS estimates of α_1 and R^2. The numbers in parentheses in the column headed by "Data (α_1)" represent the 90% confidence interval of the estimated coefficient. The real estate return from t to $t+k$ is defined as $r_{t\to t+k} = \log(p_{t+k}/p_t)$. "Low" and "High" denote the bounds of the 68% probability interval of the simulated data from the model.

Technology shocks: prediction of real estate returns by the rent-to-price ratio at different horizons

Predictive regression: $r_{t \to t+k} = \alpha_0 + \alpha_1 \log(R_{ct}/p_t) + \varepsilon_{t+k}$						
Horizon	Data (α_1)	Model (α_1)	Data (R^2)	Model (R^2)		
Quarter (year)		Median		Median	Low	High
8 (2)	0.20 (0.07, 0.33)	-0.26	0.08	0.02	0.00	0.09
12 (3)	0.37 (0.20, 0.54)	-0.34	0.15	0.03	0.00	0.10
16 (4)	0.58 (0.39, 0.78)	-0.41	0.26	0.03	0.00	0.12
20 (5)	0.77 (0.58, 0.96)	-0.46	0.40	0.03	0.00	0.13
24 (6)	0.82 (0.65, 1.00)	-0.48	0.50	0.04	0.01	0.14

Note: We report the OLS estimates of α_1 and R^2. The numbers in parentheses in the column headed by "Data (α_1)" represent the 90% confidence interval of the estimated coefficient. The real estate return from t to $t+k$ is defined as $r_{t \to t+k} = \log(p_{t+k}/p_t)$. "Low" and "High" denote the bounds of the 68% probability interval of the simulated data from the model.

Volatilities explained by discount-rate shocks (%)

Description	Volatility	Data	Model			
			Explained (%)	Median	Low	High
Investment	*std* $(\Delta \log I_t)$	1.679	48.1	0.808	0.734	0.884
Output	*std* $(\Delta \log Y_t)$	0.697	25.1	0.175	0.159	0.191
Consumption	*std* $(\Delta \log C_t)$	0.444	11.9	0.053	0.045	0.061
Rental price	*std* $(\Delta \log R_{ct})$	1.245	6.7	0.084	0.077	0.091
Real estate price	*std* $(\Delta \log p_t)$	4.171	93.7	3.910	3.611	4.193
Price-rent ratio	*std* $(\Delta \log(p_t/R_{ct}))$	3.909	100	3.923	3.625	4.211

Note: "Low" and "High" denote the bounds of the 68% probability interval of the simulated data from the model.

In contrast, the median standard deviation for $\log \theta_t$ is 0.00066 and the 0.68 probability interval is [0.00046, 0.001].

Proposition 3

- For the log-linearized model, we decompose the variance of real estate prices along the lines of Campbell and Shiller (1988) into three components.
- Iterating forward, we break down the real estate price into three components as

$$\hat{p}_t = \hat{p}_{1t} + \hat{p}_{2t} + \hat{p}_{3t},$$

$$\hat{p}_{1t} = E_t\left(\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1} - \hat{\Lambda}_t\right) + \beta E_t \hat{p}_{1t+1},$$

$$\hat{p}_{2t} = \frac{\beta(\tilde{R}_c/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t \hat{R}_{ct+1} + \beta E_t \hat{p}_{2t+1},$$

$$\hat{p}_{3t} = \frac{\lambda(1-\beta)(\tilde{p}/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t\left[\lambda \hat{p}_{t+1} - \frac{\chi}{\eta}\frac{(1+\mu)^2}{\mu^2}\left(\hat{b}_{t+1} - \hat{Y}_{t+1}\right)\right] + \beta E_t \hat{p}_{3t},$$

where $\eta = \frac{\chi(1+\mu)}{\mu-\chi}$. We can show $\eta = \frac{a^* f(a^*)}{1-F(a^*)}$.

  - The first component $\hat{p}_{1t}$: the contribution from the stochastic discount factor (discount-rate factor).
  - The second component $\hat{p}_{2t}$: the contribution from rents.
  - The third component $\hat{p}_{3t}$: the contribution from collateral values.

Variance decompositions from the discount-rate shock

- The variance of real estate prices can be decomposed into

$$
\begin{aligned}
var(\hat{p}_t) = {} & var(\hat{p}_{1t}) + var(\hat{p}_{2t}) + var(\hat{p}_{3t}) \\
& + 2cov(p_{1t}, p_{2t}) + 2cov(p_{1t}, p_{3t}) + 2cov(p_{2t}, p_{3t}).
\end{aligned}
$$

- From the simulated data with discount-rate shocks, we have $var(\hat{p}_t) = 3.910$, $var(\hat{p}_{1t}) = 2.455$, $var(\hat{p}_{2t}) = 0.104$, and $var(\hat{p}_{3t}) = 1.360$.
  - Thus, the covariance terms are unimportant.
- When the covariance terms are insignificant, we can calculate variance decompositions as

$$
var(\hat{p}_{\ell t})/var(\hat{p}_t), \text{ for} \ell = 1, 2, 3.
$$

  - The variance contribution from collateral values is more than one third:

$$
\frac{var(\hat{p}_{3t})}{var(\hat{p}_t)} = \frac{1.360}{3.910} = 0.348.
$$

Asset-price volatilities explained by discount-rate shocks (%) with no financial frictions

- Counterfactual economy with no financial frictions

Description	Volatility	Data	Model			
			Explained (%)	Median	Low	High
Real estate price	$std\,(\Delta \log p_t)$	4.171	62.5	2.607	2.409	2.796
Price-rent	$std\,(\Delta \log(p_t/R_{ct}))$	3.909	63.2	2.472	2.285	2.656

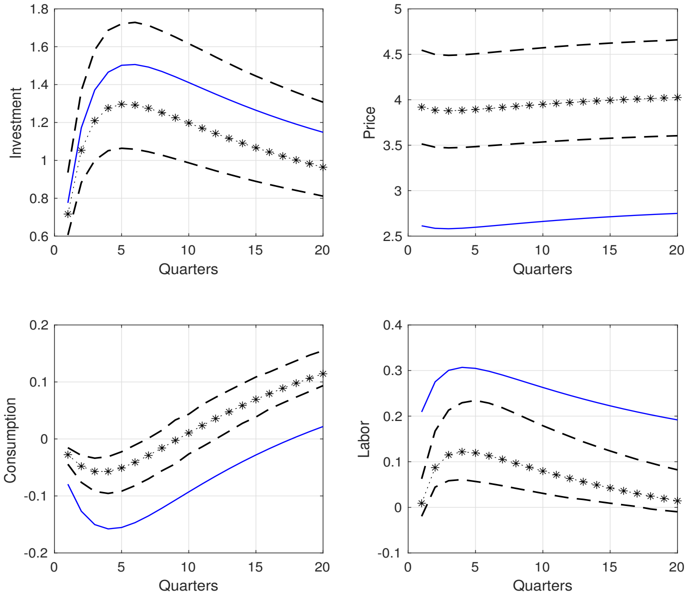
Note: "DR" stands for discount rate; "Low" and "High" denote the bounds of the 68% probability interval of the simulated data from the model.

- The absence of the contribution from collateral values is the main reason for the only 62.5% explanation of the volatility in the real estate price.

Volatilities explained by technology shocks (%)

Description	Volatility	Data	Model			
			Explained (%)	Median	Low	High
Investment	$std\,(\Delta \log I_t)$	1.679	36.2	0.607	0.537	0.678
Output	$std\,(\Delta \log Y_t)$	0.697	48.2	0.336	0.289	0.385
Consumption	$std\,(\Delta \log C_t)$	0.444	73.9	0.328	0.292	0.365
Rental price	$std\,(\Delta \log R_{ct})$	1.245	27.1	0.337	0.290	0.386
Real estate price	$std\,(\Delta \log p_t)$	4.171	12.7	0.531	0.490	0.571
Price-rent	$std\,(\Delta \log(p_t/R_{ct}))$	3.909	11.2	0.436	0.402	0.469

Note: "Low" and "High" denote the bounds of the 68% probability interval of the simulated data from the model.



Impulse responses to a one-standard-deviation shock to discount rates. The star line represents the estimated response. The dashed lines represent the 0.90 probability error bands. The solid line represents the counterfactual response for an economy without financial frictions.

Amplification and predictability: discount-rate versus technology shocks

- To understand how different shocks amplify the fluctuations of financial and real variables and generate the long-term predictability of real estate prices, consider a simplified version of our model.
  - Households maximize $E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t (\log C_t)$ subject to $C_t = w_t + D_t$. Thus, $\Lambda_t = \frac{\Theta_t}{C_t}$.
  - The intermediate-goods producer's problem:

$$\max_{N_t(j), H_t(j)} P_{Xt}(j) X_t(j) - w_t N_t(j) - R_{ct} H_t(j),$$

    subject to $X_t(j) \equiv A_t H_t^{\alpha}(j) N_t^{1-\alpha}(j)$.
  - Final-goods firms' problem remains the same.
- With this simplified model, one can obtain a closed form solution to the log-linearized model.
- We focus on two shocks: the technology shock $\hat{A}_t$ and the discount-rate shock $\hat{\theta}_t$.

Proposition 4

The log-linearized solutions for $\hat{a}_t^*$, $\hat{Y}_t$, $\hat{p}_t$, and $\hat{R}_{ct}$ are

$$\hat{a}_t^* = \frac{\chi}{\eta}\frac{1+\mu}{\mu}\frac{\rho_\theta}{1-\rho_\theta\kappa}\hat{\theta}_t,$$
$$\hat{Y}_t = \hat{A}_t + \frac{\eta\mu}{1+\mu}\hat{a}_t^*,$$
$$\hat{p}_t = \hat{A}_t + [\eta+1]\,\hat{a}_t^*,$$
$$\hat{R}_{ct} = \hat{A}_t + \hat{a}_t^*,$$

where

$$\kappa = 1-(1-\beta)(1-\lambda)-\chi(1-\beta)(1-\lambda)\left(1-\frac{1+\mu}{\mu\eta}\right)-\lambda(1-\beta)\frac{\chi}{\eta}\frac{(1+\mu)^2}{\mu^2}.$$

Relative volatility

We have the following results from Proposition 4.

- In response to a discount-rate shock,
  - $\frac{std(\hat{p}_t)}{std(\hat{R}_{ct})} = 1 + \eta$,
  - $\frac{std(\hat{p}_t - \hat{R}_{ct})}{std(\hat{Y}_t)} = \eta$.
- Thus, the real estate price is always more volatile than the rental price.
- As long as $\eta > 1$, the price-rent ratio is more volatile than output.
- By contrast, in response to a technology shock,
  - $\frac{std(\hat{p}_t)}{std(\hat{R}_{ct})} = 1$,
  - $\frac{std(\hat{p}_t - \hat{R}_{ct})}{std(\hat{Y}_t)} = 0$.
- Thus, the real estate price and the rental price fluctuate in the same magnitude.

Proposition 5

Denote the log value of the valuation ratio by $\hat{v}_t \equiv \hat{R}_{ct} - \hat{p}_t$ and the h-period return of real estate by $\hat{r}_{t+h} \equiv \hat{p}_{t+h} - \hat{p}_t$.

- In response to a discount-rate shock, we have
  - $E\left[\hat{r}_{t+h} \mid \hat{v}_t\right] = \left(1 - \rho_\theta^h\right) \frac{\eta}{1+\eta} \hat{v}_t$
  - and $R_{r,v}^2 = \frac{1}{2}\left(1 - \rho_\theta^h\right)$.
- In response to a technology shock, $E\left[\hat{r}_{t+h} \mid \hat{v}_t\right] = 0$. Thus, there is no predictability for the asset-price time series generated by technology shocks.

- The simplified model excludes investment and cannot explain the business cycle.
- But, the intuition delivered by the closed form solution helps explain the empirical results we have obtained in this paper.

Conclusion

- We build and estimate a general equilibrium model with a production economy to account for the observed volatility and predictability of real estate prices.
- The general equilibrium framework offers a concrete step toward the goal of synthesizing the analysis of asset pricing with research in the real business cycle as well as policy analysis.
- For example, one can extend our model to incorporating
    - home prices in the household sector,
    - the stock market,
    - and monetary policy.

References I

Aguiar, Mark and Gita Gopinath (2007). "Emerging Market Business Cycles: The Cycle Is the Trend". In: *Journal of Political Economy* 115.1, pp. 69–102.

Albuquerque, Rui et al. (2016). "Valuation Risk and Asset Pricing". In: *Journal of Finance* 71.6, pp. 2861–2904.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework". In: *The Handbook of Macroeconomics*. Ed. by John B. Taylor and Michael Woodford. Vol. 1C. Amsterdam, Netherlands: Elsevier. Chap. 21, pp. 1341–1393.

Campbell, John Y. and Robert J. Shiller (1988). "Stock Prices, Earnings, and Expected Dividends". In: *Journal of Finance* 43.3, pp. 661–676.

Chaney, Thomas, David Sraer, and David Thesmar (2012). "The Collateral Channel: How Real Estate Shocks affect Corporate Investment". In: *American Economic Review* 102.6, 2381–2409.

References II

Christiano, Lawrence J., Martin S. Eichenbaum, and Charles L. Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy". In: *Journal of Political Economy* 113, pp. 1–45.

Cochrane, John H. (2011). "Presidential Address: Discount Rates". In: *The Journal of Finance* 66.4, pp. 1047–1108.

Eggertsson, Gauti B. and Michael Woodford (2003). "Zero Bound on Interest Rates and Optimal Monetary Policy". In: *Brookings Papers on Economic Activity* 1, pp. 139–233.

Galí, Jordi (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications*. Second. ISBN-10: 0691164789 ISBN-13: 978-0691164786. Princeton, New Jersey: Princeton University Press.

Ghysels, Eric et al. (2013). "Forecasting Real Estate Prices". In: *Handbook of Economic Forecasting* 2. DOI: 10.1016/B978-0-444-53683-9.00009-8, pp. 509–580.

References III

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell (1997). "Long-Run Implications of Investment-Specific Technological Change". In: *American Economic Review* 87, pp. 342–362.

Hall, Robert E. (2017). "High Discounts and High Unemployment". In: *American Economic Review* 107.2. DOI: 10.1257/aer.20141297, pp. 305–330.

Iacoviello, Matteo (2005). "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle". In: *American Economic Review* 95.3, pp. 739–764.

Iacoviello, Matteo and Stefano Neri (2010). "Housing Market Spillovers: Evidence from an Estimated DSGE Model". In: *American Economic Journal: Macroeconomics* 2, pp. 125–164.

Kaplan, Greg, Kurt Mitman, and Gianluca Violante (2016). "Consumption and House Prices in the Great Recession: Model Meets Evidence". Unpublished Manuscript.

References IV

Kiyotaki, Nobuhiro and John Moore (1997). "Credit Cycles". In: *Journal of Political Economy* 105.2, pp. 211–248.

Krusell, Per et al. (2000). "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis". In: *Econometrica* 68.5, pp. 1029–1053.

Liu, Zheng, Pengfei Wang, and Tao Zha (2013). "Land-Price Dynamics and Macroeconomic Fluctuations". In: *Econometrica* 81.3, pp. 1147–1184.

Moll, Benjamin (2014). "Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?" In: *American Economic Review* 104.10, pp. 3186–3221.

Supplemental Materials

Sample correlations between real estate returns and fundamentals

- Between year-over-year real estate return and year-over-year investment growth:
  - 0.52 with the .90 probability interval (0.36, 0.64).
- Between year-over-year real estate return and year-over-year consumption growth:
  - 0.50 with the .90 probability interval (0.34, 0.63).

Historical correlations implied from the estimated model

We use the posterior mode to estimate the sequences of discount-rate shocks and permanent technology shocks over the sample.

- We obtain the historical paths of real estate price, investment, and consumption implied by the sequence of discount-rate shocks and calculate the correlations.
  - The correlation between year-over-year real estate return and year-over-year investment growth is 0.94.
  - The correlation between year-over-year real estate return and year-over-year consumption growth is -0.77. This negative correlation, however, is not economically important because the impact on consumption of a discount-rate shock is very small relative to the impact of a technology shock.
  - Because discount-rate shocks exert considerable influence on investment, the sample correlation between real estate return and investment growth is mainly driven by these shocks.

Correlations implied by the sequence of technology shocks

- We obtain the historical paths of real estate price, investment, and consumption implied by the sequence of permanent technology shocks and calculate the correlations.
  - The correlation between year-over-year real estate return and year-over-year investment growth is -0.59.
  - This negative correlation, however, is not economically important because the impact on real estate price of a technology shock is very small relative to the impact of a discount-rate shock.
  - The correlation between year-over-year real estate return and year-over-year consumption growth is insignificant (-0.06).
  - But the correlation between year-over-year investment and consumption growth rates is strong (as high as 0.82), affirming the comovement between investment and consumption.
  - This positive correlation is economically significant because the impacts of a technology shock on both investment and consumption are large as shown in both impulse responses and volatility measures.

Real estate return and consumption

How do we explain the sample correlation between real estate return and consumption growth?

- Discount-rate shocks are a primary driver of the comovement between real estate price and investment.
- Technology shocks drive most of the fluctuation of consumption and are thus a main driver of the comovement of investment and consumption.
- These two shocks combined would lead to the observed correlation between real estate return and consumption growth.
  - Conditional on the estimated sequences of discount-rate and permanent technology shocks, we use the model to generate the historical paths of real estate price, investment, and consumption, which imply
    - the 0.53 correlation between year-over-year real estate return and year-over-year investment growth
    - and the 0.41 correlation between year-over-year real estate return and year-over-year consumption growth.

What does our simple model accomplish?

- The simple model gives a closed-form solution.
- The solution helps us to understand how and why the volatility and predictability are driven by discount-rate shocks, not by technology shocks.
- While the simple model provides the closed-form solution and the associated intuition, it fails in two fundamental dimensions:
  - the comovement between real estate price and investment,
  - and the comovement between consumption and investment.
- Because investment is absent in the simple model, it fails to predict the correlation between real estate price and consumption as well.

Risk-aversion shocks

- In the home-price model of Liu, Wang, and Zha (2013), shocks to risk aversion enter an equation in which
  - the rent is equal to the marginal rate of substitution (MRS) between consumption and housing services,
  - and risk aversion enters the MRS equation.
- If risk aversion is time varying, rent would be equally volatile.
  - But rent is much smoother than house price.
  - In addition, time-varying risk aversion affects macro quantities different from discount-rate shocks.
- In the production economy, rent of commercial real estate is determined by the marginal product of real estate in the intermediate-goods production. Thus, the reasons for rejecting risk-aversion shocks are very different.

Risk-aversion shocks in the commercial real estate market

- Time-varying risk aversion may generate some movements in the price-rent ratio.
  - But it depends on how we model risk aversion.
  - Consider the utility $log(c - x)$, for example. If x increases the risk aversion, the price-rent ratio would also move.
  - The risk coefficient in the EZ preference, however, will not matter in our log-linearized model.

EpsteinZin (EZ) preferences

- The risk aversion parameter matters only in the linearized model used by the Campbell method, which takes account of the Jensen inequality term.
  - Bansal and Yaron use the Campbell linearization.
  - But the usual (log)linearization in the macro literature does not work.
  - The Tallarini JME paper has a good summary of these points.
- In general, the risk aversion parameter in the EZ preferences do not solve the asset-pricing puzzle in DSGE models.
- Rudebusch and Swanson shows the disconnection between asset prices and the real economy in their DSGE model.
  - The rise-aversion coefficient is as high as 50.
  - Even with this high value, the disconnection still exists.
- Cochrane (2011) and the Tallarini JME paper survey this literature.
- Galí (2015) explicitly introduces a discount-rate shock.
  - Our paper is the first that studies this shock's implications on price-rent dynamics.

Persistent idiosyncratic productivity shocks

- When these shocks are persistent, aggregation is very difficult, making estimation an infeasible task.
  - Even if we can solve the aggregation problem, these shocks do not generate the price-rent volatility.
  - As Cochrane (2011) argues, we need discount rates to vary.